

2.6 EULER'S METHOD

eg. $\frac{dy}{dx} = 2 - \frac{y}{x}$, $y(1) = 3$

ESTIMATE $y(2)$

LET $h = 0.1 (= \Delta x)$

$$\begin{aligned} y(1.1) &\approx y(1) + y'(1)(1.1-1) = 3 + \left(2 - \frac{3}{1}\right)(0.1) = 2.9 \\ y(1.2) &\approx y(1.1) + y'(1.1)(0.1) = 2.9 + \left(2 - \frac{2.9}{1.1}\right)(0.1) = 2.836 \\ y(1.3) &\approx y(1.2) + y'(1.2)(0.1) = 2.836 + \left(2 - \frac{2.836}{1.2}\right)(0.1) \\ &= 2.8 \end{aligned}$$

ACTUAL SOLN

$$y = x + \frac{2}{x}$$

$$y(2) = 2 + \frac{2}{2} = 3$$

IN TI-84: $1 \rightarrow X : 3 \rightarrow Y : 0.1 \rightarrow H$

$$Y + (2 - Y/X)H \rightarrow Y : X + H \rightarrow X : Y$$

$$y(2) \approx 2.947368421$$

GIVEN $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$ WITH $\Delta x = h$

NO PROGRAM

EULER'S
METHOD

IN TI-84:

$$X_0 \rightarrow X : Y_0 \rightarrow Y : \Delta x \rightarrow H$$

$$Y + (f(x, Y))H \rightarrow Y : X + H \rightarrow X : Y$$

ENTER
ENTER...

2.2 SEPARABLE 1st ORDER ODE

CAN BE WRITTEN IN FORM $\frac{dy}{dx} = g(x)p(y)$.

TO SOLVE:

$$\int \frac{1}{p(y)} dy = \int g(x) dx$$

$$P(y) = G(x) + C$$

IF POSSIBLE, SOLVE EXPLICITLY
FOR y

IS $y^2 + y' = \frac{y+1}{xy}$ SEPARABLE? PROBABLY NOT

$$\begin{aligned} y' &= \frac{y+1}{xy} - y^2 \\ &= \frac{y+1-xy^3}{xy} \end{aligned}$$

$$(xy^2 + 3y^2)dy - 2x dx = 0, \quad y(-2) = 2$$

$$(xy^2 + 3y^2) \frac{dy}{dx} - 2x = 0$$

$$\frac{dy}{dx} = \frac{2x}{xy^2 + 3y^2} = \frac{2x}{y^2(x+3)} = \underbrace{\frac{2x}{x+3}}_{g(x)} \cdot \underbrace{\frac{1}{y^2}}_{p(y)}$$

$$\int y^2 dy = \int \frac{2x}{x+3} dx$$

$$\frac{1}{3}y^3 = \int \left(2 - \frac{6}{x+3}\right) dx$$

$$\frac{1}{3}y^3 = 2x - 6 \ln|x+3| + C$$

$$\begin{array}{r} x+3 \overline{) 2x} \\ \underline{-2x-6} \\ -6 \end{array}$$

$$2 - \frac{6}{x+3}$$

$$y =$$